Lepton number violation interactions
and their effects on neutrino oscillation experiments

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Abstract

Mixing between bosons that transform differently under the standard model gauge group, but identically under its unbroken subgroup, can induce interactions that violate the total lepton number. We discuss four-fermion operators that mediate lepton number violating neutrino interactions both in a model-independent framework and within supersymmetry (SUSY) without R-parity. The effective couplings of such operators are constrained by: i) the upper bounds on the relevant elementary couplings between the bosons and the fermions, ii) by the limit on universality violation in pion decays, iii) by the data on neutrinoless double beta decay and, iv) by loop-induced neutrino masses. We find that the present bounds imply that lepton number violating neutrino interactions are not relevant for the solar and atmospheric neutrino problems. Within SUSY without R-parity also the LSND anomaly cannot be explained by such interactions, but one cannot rule out an effect model-independently. Possible consequences for future terrestrial neutrino oscillation experiments and for neutrinos from a supernova are discussed.

I. INTRODUCTION

Experimentalists have reported three different kinds of "neutrino anomalies", which seem to indicate that the standard model (SM) description of the neutrino is incorrect. Today, many physicists consider the recent SuperKamiokande high statistics result [1], which confirmed the long-standing atmospheric neutrino (AN) problem [2], as the strongest experimental evidence for New Physics (NP) beyond the SM. However, also increasingly convincing arguments, that the solar neutrino (SN) data can only be explained by extending the SM neutrino picture, have been established in recent years [3]. Finally the LSND collaboration has found unexpected signals for neutrino flavor conversion in two appearance experiments [4,5]. So far none of the other short baseline experiments [6–8], has been able to confirm these results, but major experimental efforts are underway to search for neutrino oscillations both at short [9] and long baseline [10–13] facilities.

The favorite explanation for the existing neutrino anomalies is to allow for massive neutrinos that mix and therefore undergo flavor oscillations while propagating. Neutrino oscillations provide convincing solutions to each of the above mentioned neutrino problems. However, the SN, the AN and the LSND observations imply three separated scales for the mass-squared differences $\Delta_{ij} = m_i^2 - m_j^2$

$$\Delta_{SN} \lesssim 10^{-5} \, \text{eV}^2 \,, \tag{1.1}$$

$$\Delta_{AN} \sim 10^{-3} \, \text{eV}^2 \,, \tag{1.2}$$

$$\Delta_{LSND} \gtrsim 10^{-1} \, \text{eV}^2 \,, \tag{1.3}$$

which cannot be accommodated simultaneously in a three neutrino framework [14]. Consequently, unless one ignores one of the three anomalies or allows for a forth non-sequential light neutrino [15], already the present neutrino data indicate that neutrino masses and mixing alone might not be the complete picture of the New Physics in the neutrino sector. It is important to note that many extensions of the SM that could provide massive neutrinos also predict non-standard neutrino interactions. In fact, in some cases new interactions induce neutrino masses in loop processes [16] and one can relate the two aspects of New Physics quantitatively.

Solutions of the various neutrino anomalies in terms of new interactions with and without neutrino masses and mixing have been studied in Ref. [17–23]. While for the SN problem this is indeed a viable possibility [22,23] it has been shown that new flavor changing neutrino interactions that conserve total lepton number are constrained by the high precision data that confirm the SM predictions to be too small to affect the atmospheric [24] and the LSND [25] anomalies.

In this work we study another class of NP interactions, which so far has only received little attention [20], namely new neutrino interactions that violate the total lepton number L. Such interactions can arise naturally in models where there is mixing between bosons that

transform differently under the SM gauge group, but identically under its unbroken subgroup. As an example consider the anomalous muon decays that produce two antineutrinos

$$\mu^+ \to e^+ \bar{\nu}_e \bar{\nu}_\ell \qquad (\ell = e, \mu, \tau). \tag{1.4}$$

Such decays violate L by two units. In principle the reaction in (1.4) could produce the $\bar{\nu}_e$'s that are observed at LSND in the decay at rest (DAR) channel and which are usually accounted for by $\bar{\nu}_\mu \to \bar{\nu}_e$ flavor oscillations.

Unlike for the L-conserving interactions, replacing the antineutrinos by their (positively) charged $SU(2)_L$ partners gives rise to interactions that violate $U(1)_{EM}$. Thus, it follows that the effective couplings of the above decays (1.4) must vanish in the $SU(2)_L$ symmetric limit and be proportional to $SU(2)_L$ breaking effects. This breaking is not proportional to a mass splitting within a given multiplet as for the L-conserving interactions [25], but it shows up as *mixing* between (heavy) bosons of different $SU(2)_L$ representations.

In Section II we present the general framework that expresses the effective strength of the lepton number violating interactions (as well as those that conserve total lepton number) in terms of the boson masses, their mixing angle and the relevant trilinear couplings. In Section III we discuss supersymmetry without R-parity (SUSY $\not{R}_p$) as a prominent example for such a scenario, where the mixing between left-handed and right-handed sfermions that couple to the SM fermions via $\not{R}_p$ violating interactions, induces lepton number violating interactions. In Section IV we establish relations between lepton number violating interactions and those that conserve total lepton number. We use these relations to derive constraints on the new interactions. Additional bounds on these interactions arise from the limit on universality violation in pion decays, the data on neutrinoless double beta decay and from loop-induced neutrino masses. In Section V we investigate whether the lepton number violating interactions could be relevant for any of the three anomalies as well as for the up-coming terrestrial neutrino oscillation experiments. Also implications for neutrinos from a supernova are discussed. We conclude in Section VI.

II. FORMALISM

Consider a generic extension of the standard model with two bosonic fields ϕ and χ that transform differently under $SU(2)_L$. In general, after $SU(2)_L$ breaking, the $SU(2)_L$ components of these fields ϕ_q, χ_q which transform identically under the unbroken SM gauge group $SU(3)_C \times U(1)_{EM}$ can mix with each other giving rise to a hermitian mass-matrix

$$\boldsymbol{M}^2 = \begin{pmatrix} M_{11}^2 & M_{12}^2 \\ M_{21}^2 & M_{22}^2 \end{pmatrix}. \tag{2.1}$$

Diagonalizing $\boldsymbol{M}^2$ yields the eigenvalues

$$M_{1,2}^2 = \frac{1}{2} \left(\Sigma \mp \sqrt{\delta^2 + 4|M_{12}^2|^2} \right), \tag{2.2}$$

with $\Sigma = M_{11}^2 + M_{22}^2$ and $\delta^2 = (M_{11}^2 - M_{22}^2)^2$. The mass-eigenstates are linear combinations of ϕ_q and χ_q, i.e.

$$|i\rangle = V_{i1}|\phi_q\rangle + V_{i2}|\chi_q\rangle\,, \qquad (i = 1, 2)\,. \tag{2.3}$$

Assuming that $M_{12} = M_{21}$ is real, the mixing matrix can be parameterized as

$$\boldsymbol{V} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\,, \tag{2.4}$$

with

$$\sin 2\theta = \frac{2M_{12}^2}{\sqrt{\delta^2 + 4M_{12}^4}}. \tag{2.5}$$

Let us add now renormalizable interactions that couple the bosonic fields ϕ and χ to bilinears A, B which are built out of two SM fermions

$$-\mathcal{L}_{A,B} = \lambda_A\,(\phi\,A) + \lambda_B\,(\chi\,B) + \text{h.c.}\,, \tag{2.6}$$

where λ_A and λ_B denote the elementary trilinear couplings. These couplings are induced by New Physics that may be present at or above the weak scale. Any such theory will include the SM gauge symmetry, implying that $\mathcal{L}_{A,B}$ is invariant under $\mathcal{G}_{\text{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y$. If the bosons are vector fields the couplings in (2.6) may be gauge interactions. We will focus on scalar bosons that couple to fermions via *a priori* arbitrary Yukawa couplings $\lambda_{A,B}$. The fermion bilinears may be composed of quarks, leptons or both, as well as the respective antiparticles, which all belong to either $SU(2)_L$ singlets or doublets. Then gauge invariance implies that the bosonic fields may be singlets (s), doublets (d) or triplets (t) of $SU(2)_L$. Since we require ϕ and χ to have different transformation properties under $SU(2)_L$ also A and B will transform differently. On the other hand, since ϕ and χ transform identically under $SU(3)_C$ this also applies to A and B.

Given the elementary couplings in (2.6) one can construct four-fermion interactions that are mediated by the bosonic fields. Each bilinear A and B can be either coupled to itself or there can be a coupling *between* A and B. Since ϕ_q and χ_q are required to have the same electric charge q there is only a coupling between the $SU(2)_L$ components A_q and B_q that have the same charge such that the resulting four-fermion operator $A_q^\dagger B_q$ conserves $U(1)_{EM}$. Similarly, since A and B transform identically under $SU(3)_C$ it follows that $A_q^\dagger B_q$ is a color singlet.

We stress that the coupling between fermion bilinears that have different $SU(2)_L$ transformations requires the mixing between ϕ_q and χ_q. As a unique consequence the coupling between a bilinear that transforms as an $SU(2)_L$ doublet to a bilinear that is a singlet or a triplet of $SU(2)_L$ can induce effective four-fermion operators that do not conserve the total lepton number L. In addition such operators may also violate the individual lepton number L_ℓ. For example, if the $SU(2)_L$ doublet $A = \bar{L}_e E_\mu$ (E_ℓ and L_ℓ denote a lepton singlet and

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doublet field, respectively, of flavor ℓ) couples to an $SU(2)_L$ doublet ϕ and if the $SU(2)_L$ singlet $B_{-1} = (L_\mu L_e)_s$ couples to an $SU(2)_L$ singlet χ^+, then the mixing between the $q = 1$ doublet-component ϕ^+ and χ^+ gives rise to the operator $A^\dagger_{-1} B_{-1} = (\overline{\mu_R} \nu_e)(\nu_\mu e_L - \mu_L \nu_e)$, which induces $\mu^+_L \to e^+_R \bar{\nu}_\mu \bar{\nu}_e$ and $\mu^+_L \nu_e \to \mu^+_R \bar{\nu}_e$. Both processes violate L_e, L_μ and L by two units.

Note that if one scalar field couples to two different bilinears (which, consequently must have the same $SU(2)_L$ transformation) then these two bilinears can be coupled to each other. However, the four fermion operators that arise from this mechanism (see e.g. [25]) may only violate L_ℓ, but not L.

The effective four-fermion operators $A^\dagger_q A_q$, $B^\dagger_q B_q$ and $A^\dagger_q B_q$ at energies well below the masses of the scalar fields [i.e. the eigenvalues of $\boldsymbol{M}^2$ given in (2.2)] are obtained by integrating out the bosonic degrees of freedom. Assuming weak trilinear couplings, $\lambda_{A,B} \lesssim 1$, the tree-level diagrams result into the effective couplings

$$G^{A^\dagger A}_N = \frac{|\lambda_A|^2}{4\sqrt{2}M^2_A}, \qquad G^{B^\dagger B}_N = \frac{|\lambda_B|^2}{4\sqrt{2}M^2_B}, \qquad G^{A^\dagger B}_N = \frac{\lambda^*_A \lambda_B}{4\sqrt{2}M^2_{AB}}, \tag{2.7}$$

where the respective low-energy propagators are given by

$$M^{-2}_A \equiv \sum_i \frac{\langle\phi|i\rangle\langle i|\phi\rangle}{M^2_i} = \frac{\cos^2\theta}{M^2_1} + \frac{\sin^2\theta}{M^2_2}, \tag{2.8}$$

$$M^{-2}_B \equiv \sum_i \frac{\langle\chi|i\rangle\langle i|\chi\rangle}{M^2_i} = \frac{\cos^2\theta}{M^2_2} + \frac{\sin^2\theta}{M^2_1}, \tag{2.9}$$

$$M^{-2}_{AB} \equiv \sum_i \frac{\langle\phi|i\rangle\langle i|\chi\rangle}{M^2_i} = \frac{\sin 2\theta}{2}\left(\frac{1}{M^2_1} - \frac{1}{M^2_2}\right). \tag{2.10}$$

From (2.10) it is obvious that $A^\dagger_q$ and B_q can only couple to each other when there is a non-vanishing mixing ($\sin 2\theta \neq 0$) and when the physical masses are not degenerate ($M_1 \neq M_2$). We note that for maximal mixing ($\sin 2\theta = 1$) the propagator M^{-2}_A equals to M^{-2}_B, and M^{-2}_{AB} is maximal. Moreover, we remark that using (2.5) it follows that the propagator in (2.10) is simply

$$M^{-2}_{AB} = \frac{M^2_{12}}{M^2_1 M^2_2}. \tag{2.11}$$

III. LEPTON NUMBER VIOLATION IN SUSY WITHOUT R-PARITY

In this section we present an explicit example for the general mechanism developed in Section II by discussing scalar mixing in supersymmetric extensions of the standard model without R-parity [26]. In particular, we show how the model-specific parameters of this theory translate into those we introduced in Section II.

The R-parity violating couplings $\lambda_{ijk}L_iL_jE_k^c$ and $\lambda'_{ijk}L_iQ_jD_k^c$, where $L_\kappa, Q_\kappa, E_\kappa$ and D_κ denote the chiral superfields containing, respectively, the left-handed lepton and quark doublets and the right-handed charged-lepton and d-quark singlets of generation $\kappa = 1, 2, 3$, introduce a variety of couplings between fermion bilinears and sfermions:

$$-\mathcal{L}_\lambda = \lambda_{ijk}\left[\tilde{\nu}_L^i\overline{\ell_R^\kappa}\ell_L^j + \tilde{\ell}_L^j\overline{\ell_R^\kappa}\nu_L^i + \tilde{\ell}_R^{\kappa*}\overline{\nu_L^{ic}}\ell_L^j - (i \to j)\right] + \text{h.c.}, \tag{3.1}$$

$$-\mathcal{L}_{\lambda'} = \lambda'_{ijk}\left[\tilde{\nu}_L^i\overline{d_R^\kappa}d_L^j + \tilde{d}_L^j\overline{d_R^\kappa}\nu_L^i + \tilde{d}_R^{\kappa*}\overline{\nu_L^{ic}}d_L^j - \tilde{e}_L^i\overline{d_R^\kappa}u_L^j - \tilde{u}_L^j\overline{d_R^\kappa}e_L^i - \tilde{d}_R^{\kappa*}\overline{e_L^{ic}}u_L^j\right] + \text{h.c.}. \tag{3.2}$$

Due to charge conservation only sfermions of the same type can mix. In principle mixing is allowed between the left-handed and the right-handed components of the (charged) sfermions, as well as between sfermions of different generation, but for simplicity we will assume that the latter is negligible.

The leptonic couplings in (3.1) can induce L-violating interactions like in (1.4) [20]. For example, identifying the scalar fields $\phi^+ = \tilde{\tau}_R^+$ and $\chi^+ = \tilde{\tau}_L^+$ and the couplings $\lambda_A = \lambda_{132}^*$ and $\lambda_B = \lambda_{123}$ reproduces exactly the example in Section II that gave rise to $\mu_L^+ \to e_R^+ \bar{\nu}_\mu \bar{\nu}_e$. Note that SUSY without R_p not only provides the required scalar fields and their couplings, but also an explicit expression for the mass-matrix in (2.1), i.e. [27]

$$\boldsymbol{M}_f^2 = \begin{pmatrix} M_{\tilde{L}}^2 + m_f^2 + M_Z^2(T_3^f/2 - q_f\sin^2\theta_W) & m_f(A_f - \mu\cot^{T_3^f}\beta) \\ m_f(A_f - \mu\cot^{T_3^f}\beta) & M_{\tilde{R}}^2 + m_f^2 + M_Z^2 q_f\sin^2\theta_W \end{pmatrix}, \tag{3.3}$$

where m_f and q_f denote the mass and the charge of the fermion f, $T_3^f = 1\ (-1)$ for $f = u_\kappa\ (d_\kappa, e_\kappa)$, $M_{\tilde{L}}^2\ (M_{\tilde{R}}^2)$ is the soft supersymmetric breaking mass-squared term for the left- (right-)handed sfermion, and A_ℓ, μ and $\tan\beta$ are the familiar SUSY parameters [27].

We note that in the absence of right-handed (s)neutrinos $\tilde{f}_L - \tilde{f}_R$ mixing can occur only for charged sfermions. This implies that the mass-matrix (3.3) has to be positive definite in order to avoid the spontaneous breaking of $U(1)_{EM}$.

IV. EXPERIMENTAL CONSTRAINTS

In this section we discuss constraints on the effective couplings for lepton number violating neutrino interactions. As we mentioned already, the corresponding four-fermion operators cannot be related to the ones where the neutrinos are rotated into their charged lepton partners, since such an $SU(2)_L$ rotation violates $U(1)_{EM}$. Hence, while in many cases such a rotation can provide stringent bounds on the product of trilinear couplings for interactions that only violate L_ℓ (see [24,25,20]), it does not help for L-violating neutrino interactions. Instead one can use the constraints on each of the trilinear couplings which arise from the interactions induced by the self-couplings of a specific fermion bilinear relevant for the lepton number violating neutrino interaction. Alternatively, in some cases, there are direct constraints on the L-violating interactions. Operators that induce lepton number violating pion decays can be constrained using the limit on universality violation.

Upper bounds on certain operators containing the electron neutrino follow from the data on neutrinoless double beta decay. Finally, in case the L-violating operator involves two neutrinos, one can connect the two external charged fermions in order to generate neutrino masses and use their upper bounds.

A. Constraints from the trilinear couplings

Any non-vanishing trilinear coupling λ_A between a fermion bilinear A and a boson ϕ can be used to create the effective interaction

$$\frac{|\lambda_A|^2 A^\dagger A}{4\sqrt{2}M_A^2}. \tag{4.1}$$

If the intermediate boson does not mix, the low-energy propagator is simply $M_A^{-2} = M_\phi^{-2}$, but if there is mixing the correct expression is the one in (2.8).

This is important, since the constraints on various trilinear couplings in the literature are derived assuming that the respective intermediate boson is a mass eigenstates which has a definite mass M. Therefore, if we denote the upper bound on any trilinear coupling derived under such an assumption by $\hat{\lambda}_A$, then this implies for the parameter λ_A, which describes the coupling between A and a boson ϕ that is not a mass eigenstate (but which mixes with a different boson χ as discussed in Section II), that

$$|\lambda_A| < \hat{\lambda}_A \times \frac{M_A}{M}. \tag{4.2}$$

This rescaling corrects for the fact that if the effective propagator M_A^{-2} is smaller (larger) than M^{-2}, then the constraint on λ_A will be weaker (stronger).

Consequently the upper bound on any L-violating operator $(G_N^{A^\dagger B}/\sqrt{2})\,A^\dagger B$ which is induced by $\phi - \chi$ mixing is constrained by

$$G_N^{A^\dagger B} = \frac{\lambda_A^* \lambda_B}{4\sqrt{2}M_{AB}^2} < \frac{\hat{\lambda}_A^* \hat{\lambda}_B}{4\sqrt{2}M^2} \times \frac{M_A\,M_B}{M_{AB}^2}. \tag{4.3}$$

The upper bound on the right-hand side of (4.3) factorizes into

$$\hat{G}_N^{A^\dagger B} \equiv \frac{\hat{\lambda}_A^* \hat{\lambda}_B}{4\sqrt{2}M^2}, \tag{4.4}$$

which only depends on the upper bounds $\hat{\lambda}_A$ and $\hat{\lambda}_B$ derived from experimental observations (under the assumption that the intermediate particle has mass M) and the ratio

$$\varrho \equiv \frac{M_A\,M_B}{M_{AB}^2} \tag{4.5}$$

which is a function of the mixing angle θ and the mass eigenvalues M_1 and M_2 only. Note that $\varrho(\sin\theta, M_1, M_2) \leq 1$ and that ϱ is maximal at $\sin\theta = \cos\theta = 1/\sqrt{2}$, where it takes the

value $\hat{\varrho} = (M_2^2 - M_1^2)/(M_2^2 + M_1^2)$, which is small when the masses are almost degenerate, but it quickly approaches unity when the degeneracy is lifted. We show $\varrho(\sin^2\theta)$ for various values of M_2/M_1 in Fig. 1.

Since the interactions induced by the self-couplings of any fermion bilinear A do not violate L_ℓ and L, the corresponding NP operator only induces additional contributions to reactions that are already present in the standard model. Therefore any non-zero NP effective coupling $G_N^{A^\dagger A}$ modifies the SM predictions for the relevant processes and precision measurements can be used to put upper bounds on $G_N^{A^\dagger A}$.

It is conventional to assume that only one trilinear coupling λ_A is non-zero for each bound and that the intermediate boson has a mass of $M = 100$ GeV. Then the constraint is expressed in terms the dimensionless real number $\hat{\lambda}_A$ as

$$|\lambda_A| < \hat{\lambda}_A \times \left(\frac{M}{100 \text{ GeV}}\right), \tag{4.6}$$

which translates into

$$G_N^{A^\dagger A} < \hat{G}_N^{A^\dagger A} \equiv \frac{\hat{\lambda}_A^2}{4\sqrt{2}(100 \text{ GeV})^2} = 1.52\,\hat{\lambda}_A^2\,G_F. \tag{4.7}$$

for the effective coupling. If the process that was used to derive the bound and the one which one wants to constrain are mediated by bosons which are different members of the same $SU(2)_L$ multiplet, then one has to correct for differences in the propagators

$$G_N^{q'} = \frac{M_q^2}{M_{q'}^2} G_N^q < 1.52\,\hat{\lambda}^2\,\frac{M_q^2}{M_{q'}^2}\,G_F\,, \tag{4.8}$$

where q, q' refer to the charge of the intermediate boson. In Ref. [24] it has been shown that electroweak precision data imply that $M_q/M_{q'}$ is of order unity. Even for masses close to the weak scale this ratio is at most 2.6 unless one allows for some fine-tuned cancellations.

In Tab. 1 we list all bilinears that couple to scalar weak singlets or doublets that appear in SUSY without R-parity. We also show in Tab. 1 the upper bounds (at 2σ) for both the trilinear couplings ($\hat{\lambda}$) and the effective couplings ($\hat{G}_N^{A^\dagger A}$). We assume here that $M_q/M_{q'} = 1$, bearing in mind that for scalar doublets the maximal correction from $SU(2)_L$ breaking effects could be a factor a few. For the bounds we use the results [28] obtained within the framework of SUSY $\not{R}_p$. All limits are at 2σ, except for $\lambda_{1\kappa 1}$ which is at 3σ. The most stringent constraints relevant to our discussion arise from charged current universality (V_{ud}), lepton universality [$R_\tau = \Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$, $R_\pi = \Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ and $R_{\tau\pi} = \Gamma(\tau \to \pi\nu_\tau)/\Gamma(\pi \to \mu\nu_\mu)$], forward-backward asymmetries in e^+e^- collisions at the Z peak (A_{FB}), atomic parity violation (APV), ν_μ deep inelastic scattering (ν_μ DIS) and constraints on the compositeness scale [$\Lambda(qqqq)$]. We note that the listed bounds apply to any theory that contains the respective trilinear coupling, since we consider only the constraints that are derived directly from a specific coupling, that is we allow only one term in the R_p-violating couplings (3.1) and (3.2), to be non-zero at a time. If one relaxes

this assumption and takes all the R_p-violating couplings together as they appear in (3.1) and (3.2), i.e. one evokes supersymmetry, then in some cases additional constraints (given is square brackets) can be found from processes which have different intermediate scalars, but rely on the same trilinear coupling. We note that for the coupling λ_{31} of $L_3\overline{D}_1$ to a scalar doublet to the best of our knowledge there is no model-independent bound. Demanding that the theory remains perturbative at large energies implies that $\lambda_{31} \lesssim 1$. In SUSY $\not{R}_p$ $\lambda_{31} = \lambda'_{3\kappa 1} < 0.52$, which is due to the upper bound on λ'_{321} from D_s decays.

Tab. 1: Experimental constraints on fermion bilinear self-couplings

λ_A	A	ϕ	$\hat{\lambda}_A$ [SUSY $\not{R}_p$]	$\hat{G}_N^{A^\dagger A}/G_F$ [SUSY $\not{R}_p$]	from
$\lambda_{12\kappa}$	$L_1 L_2$	$\tilde{e}_R^\kappa$	0.05	0.0038	V_{ud}
$\lambda_{13\kappa}$	$L_1 L_3$	$\tilde{e}_R^\kappa$	0.06	0.0055	R_τ
$\lambda_{23\kappa}$	$L_2 L_3$	$\tilde{e}_R^\kappa$	0.06	0.0055	R_τ
$\lambda_{1\kappa 1}$	$L_1 \overline{E}_1$	$\tilde{\nu}_L^\kappa$	0.37 [0.06]	0.21 [0.0055]	A_{FB}
$\lambda_{2\kappa 1}$	$L_2 \overline{E}_1$	$\tilde{\nu}_L^\kappa$	0.25 [0.07]	0.095 [0.0074]	A_{FB}
$\lambda_{3\kappa 1}$	$L_3 \overline{E}_1$	$\tilde{\nu}_L^\kappa$	0.11 [0.07]	0.018 [0.0074]	A_{FB}
$\lambda_{1\kappa 2}$	$L_1 \overline{E}_2$	$\tilde{\nu}_L^\kappa$	0.25 [0.06]	0.095 [0.0055]	A_{FB}
$\lambda_{2\kappa 2}$	$L_2 \overline{E}_2$	$\tilde{\nu}_L^\kappa$	0.25 [0.06]	0.095 [0.0055]	A_{FB}
$\lambda_{3\kappa 2}$	$L_3 \overline{E}_2$	$\tilde{\nu}_L^\kappa$	0.25 [0.06]	0.095 [0.0055]	A_{FB}
$\lambda'_{11\kappa}$	$L_1 Q_1$	$\tilde{d}_R^\kappa$	0.02	0.0006	V_{ud}
$\lambda'_{21\kappa}$	$L_2 Q_1$	$\tilde{d}_R^\kappa$	0.06	0.0055	R_π
$\lambda'_{31\kappa}$	$L_3 Q_1$	$\tilde{d}_R^\kappa$	0.11	0.018	$R_{\tau\pi}$
$\lambda'_{1\kappa 1}$	$L_1 \overline{D}_1$	$\tilde{q}_L^\kappa$	0.02	0.0006	APV
$\lambda'_{2\kappa 1}$	$L_2 \overline{D}_1$	$\tilde{q}_L^\kappa$	0.22	0.07	ν_μ DIS
$\lambda'_{3\kappa 1}$	$L_3 \overline{D}_1$	$\tilde{q}_L^\kappa$	$\hat{\lambda}_{31}$ [0.52]	$1.52\,\hat{\lambda}_{31}^2$ [0.41]	"ν_τ DIS"
$\lambda'_{\kappa 11}$	$Q_1 \overline{D}_1$	$\tilde{\ell}_L^\kappa$	0.3 [0.11]	0.14 [0.018]	$\Lambda(qqqq)$

In general there could also be trilinear couplings involving the up-type quark singlet as well as couplings to scalar triplets, which we do not discuss explicitly. We remark that replacing the scalar weak singlets by triplets of the same charge, while keeping the flavor structure, only changes the sign in the doublet-doublet contraction and yields the same effective interactions. However, a neutral triplet may also couple to $\nu\nu$ inducing additional effective couplings. Moreover a triplet can have flavor diagonal coupling to LL, while for scalars λ has to be antisymmetric in flavor space. The Δ_L in left-right symmetric models is an example for a scalar triplet with flavor diagonal couplings. We do not consider here the possibility of intermediate vector bosons, which will couple to different bilinears than the scalar fields, and moreover produce a different spin structure for the four-fermion operator.

From the definition of $\hat{G}_N^{A^\dagger B}$ and $\hat{G}_N^{A^\dagger A}$ it follows that

$$\hat{G}_N^{A^\dagger B} = \sqrt{\hat{G}_N^{A^\dagger A}\,\hat{G}_N^{B^\dagger B}}. \tag{4.9}$$

In Tab. 2 we show $\hat{G}_N^{A^\dagger B}$ (based on this relation and the constraints tabulated in Tab. 1) for the various L-violating effective couplings that are relevant for neutrino oscillation experiments.

Tab. 2: Experimental constraints on L-violating couplings

$A^\dagger$	B	$\hat{G}_N^{A^\dagger B}/G_F$ [SUSY $\not{R}_p$]	reaction	relevant for
$L_1\overline{E}_2$	L_1L_2	0.019 [0.0046]	$\mu_L^+ \to e_R^+\,\bar{\nu}_e\,\bar{\nu}_\mu$	LSND: DAR
$L_1\overline{E}_2$	L_1L_3	0.023 [0.0055]	$\mu_L^+ \to e_R^+\,\bar{\nu}_e\,\bar{\nu}_\tau$	LSND: DAR
$L_1\overline{D}_1$	L_1Q_1	0.0006	$\nu_e\,u_L \to d_R\,e_R^+$	LSND: "fake" $\bar{\nu}_e$
$L_2\overline{D}_1$	L_1Q_1	0.0067	$\nu_\mu\,u_L \to d_R\,e_R^+$	LSND: "fake" $\bar{\nu}_e$
$Q_1\overline{D}_1$	L_1L_2	0.023 [0.0083]	$\nu_\mu\,u_L \to d_R\,e_R^+$	LSND: "fake" $\bar{\nu}_e$
$L_2\overline{D}_1$	L_3Q_1	0.037	$\nu_\mu\,u_L \to d_R\,\tau_R^+$	NOMAD/CHORUS: "fake" $\bar{\nu}_\tau$
$Q_1\overline{D}_1$	L_2L_3	0.028 [0.010]	$\nu_\mu\,u_L \to d_R\,\tau_R^+$	NOMAD/CHORUS: "fake" $\bar{\nu}_\tau$
$L_1\overline{E}_1$	L_1L_2	0.028 [0.0046]	$\nu_e\,e_L^- \to \bar{\nu}_\mu\,e_R^-$	SN: $\nu_e \to \bar{\nu}_\mu$
$L_1\overline{E}_1$	L_1L_3	0.034 [0.0055]	$\nu_e\,e_L^- \to \bar{\nu}_\tau\,e_R^-$	SN: $\nu_e \to \bar{\nu}_\tau$
$L_1\overline{D}_1$	L_1Q_1	0.0006	$\nu_e\,d_L \to \bar{\nu}_e\,d_R$	SN: $\nu_e \to \bar{\nu}_e$
$L_1\overline{D}_1$	L_2Q_1	0.0018	$\nu_e\,d_L \to \bar{\nu}_\mu\,d_R$	SN: $\nu_e \to \bar{\nu}_\mu$
$L_1\overline{D}_1$	L_3Q_1	0.0033	$\nu_e\,d_L \to \bar{\nu}_\tau\,d_R$	SN: $\nu_e \to \bar{\nu}_\tau$
$L_2\overline{D}_1$	L_1Q_1	0.0067	$\nu_e\,d_L \to \bar{\nu}_\mu\,d_R$	SN: $\nu_e \to \bar{\nu}_\mu$
$L_3\overline{D}_1$	L_1Q_1	$0.030\,\hat{\lambda}_{31}$ [0.016]	$\nu_e\,d_L \to \bar{\nu}_\tau\,d_R$	SN: $\nu_e \to \bar{\nu}_\tau$
$L_2\overline{E}_1$	L_1L_3	0.023 [0.0064]	$\nu_\mu\,e_L^- \to \bar{\nu}_\tau\,e_R^-$	AN: $\nu_\mu \to \bar{\nu}_\tau$
$L_3\overline{E}_1$	L_1L_2	0.0083 [0.0053]	$\nu_\mu\,e_L^- \to \bar{\nu}_\tau\,e_R^-$	AN: $\nu_\mu \to \bar{\nu}_\tau$
$L_2\overline{D}_1$	L_3Q_1	0.037	$\nu_\mu\,d_L \to \bar{\nu}_\tau\,d_R$	AN: $\nu_\mu \to \bar{\nu}_\tau$
$L_3\overline{D}_1$	L_2Q_1	$0.091\,\hat{\lambda}_{31}$ [0.047]	$\nu_\mu\,d_L \to \bar{\nu}_\tau\,d_R$	AN: $\nu_\mu \to \bar{\nu}_\tau$

From Tab. 2 one can see that model-independently almost all effective couplings for the lepton number violating operators are constrained to be at most a few percent of G_F. The weakest constraints are those involving λ_{31}, but even allowing λ_{31} to be of order unity implies that $G_N^{A^\dagger B} \lesssim 0.1\,G_F$. Imposing SUSY we find that all of the effective couplings are constrained to be less than one percent of G_F, except those involving $\lambda'_{3\kappa1}$ which could be at most a few percent of G_F.

B. Direct constraints

We turn now to a discussion of additional constraints on the effective four-fermion operators that violate total lepton number. Unlike the bounds derived in the previous section

these bounds do not depend upon the constraints on the trilinear couplings, but apply to the L-violating operator itself.

1. Pion decays

Consider the ratio between the decay rates of $\pi^+ \to e^+\nu$ and $\pi^+ \to \mu^+\nu$,

$$R_\pi = \frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)} \, . \tag{4.10}$$

The measured value of this ratio [29],

$$R_\pi(expt) = (1.235 \pm 0.004) \times 10^{-4} \, , \tag{4.11}$$

is in good agreement with the value predicted by the standard model, including radiative corrections [30],

$$R_\pi(SM) = (1.230 \pm 0.008) \times 10^{-4} \, . \tag{4.12}$$

Consequently any non-standard contribution to either $\pi^+ \to e^+\nu$ or $\pi^+ \to \mu^+\nu$ is constrained to be small [31]. Since the final neutrino is not detected this applies to π^+ decays with both final neutrinos and antineutrinos. The latter case is particularly interesting, because it allows to constrain lepton number violating operators that induce pion decays. Note that for these operators (unlike for lepton number conserving operators) the decay amplitude is not suppressed by the charged lepton mass m_ℓ $(\ell = e, \mu)$, but there is an enhancement by [31]

$$f_\ell \equiv \frac{m_\pi^2}{m_\ell \, (m_u + m_d)} \tag{4.13}$$

with respect to the standard model currents. Therefore, when adding NP interactions to those of the SM, to leading order in the effective couplings $G_N^\ell \ll G_F$ of the lepton number violating operators that induce $\pi^+ \to \ell \, \bar\nu$, the ratio in (4.10) is

$$\frac{R_\pi(SM + NP)}{R_\pi(SM)} = 1 + \frac{(f_e \, G_N^e)^2 - (f_\mu \, G_N^\mu)^2}{(V_{ud} \, G_F)^2} \, , \tag{4.14}$$

where V_{ud} is the CKM matrix element relevant for the SM pion decay. Then, assuming that there are no fine-tuned cancellations in (4.14), it follows from (4.11) and (4.12) that

$$G_N^e \lesssim 3 \times 10^{-5} \, G_F \, , \tag{4.15}$$

$$G_N^\mu \lesssim 4 \times 10^{-3} \, G_F \, . \tag{4.16}$$

We conclude that the effective couplings of all the operators in second section of Tab. 2 that induce $\nu_\ell \, u \to d \, e^+$ must be severely suppressed due to the bound on G_N^e, since

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they also give rise to the lepton number violating pion decays. In particular the model-independent bound for $(Q_1\overline{D}_1)(L_1L_2)$ is improved significantly. Moreover, also the operator $(Q_1\overline{D}_1)(L_2L_3)$ can be constrained by the limit on G_N^μ, because the structure of the singlet bilinear $(L_2L_3)_s = \nu_\mu\tau - \mu\nu_\tau$, implies that the operator obtained by exchanging the flavors of the neutrino and the charged lepton must have the same effective coupling. A similar argument applies to the operators $(L_\alpha\overline{D})(L_\ell Q)$ ($\alpha = e, \mu, \tau$ and $\ell = e, \mu$) appearing in the third and forth section of Tab. 2. Since $(L_\ell Q)_s = \ell u_L - \nu_\ell d_L$ they induce both $\nu_\ell\, d_L \to \nu_\alpha\, d_R$ and $\pi^+ \to \ell^+\,\bar{\nu}_\alpha$. However the upper bounds on G_N^ℓ in (4.15) and (4.16) are not useful to constrain any of the purely leptonic operators or those involving $(QL_3)_s$.

2. Neutrinoless double beta decay

The combination of the SM operator for beta decay with a new physics operator that mediates the L-violating process

$$\bar{\nu}_e\, n \to e^-\, p \tag{4.17}$$

gives rise to neutrinoless double beta decay ($0\nu\beta\beta$) [32] due to the exchange of a virtual neutrino. The crucial point is that if the leptonic current of the (if necessary Fierz transformed) NP operator contains a right-handed neutrino then the contribution from the neutrino propagator is

$$\propto P_L\frac{q^\mu\gamma_\mu + m_\nu}{q^2 - m_\nu^2}P_R = \frac{q^\mu\gamma_\mu}{q^2 - m_\nu^2}\,. \tag{4.18}$$

Therefore the $0\nu\beta\beta$ amplitude is proportional to $G_N\,G_F\cdot q$, where the neutrino momentum q is typically given by the nuclear Fermi momentum $p_F \simeq 100\,\mathrm{MeV}$ [32]. The present half-life limit of the Heidelberg-Moscow experiment, $T_{1/2}^{0\nu\beta\beta} > 1.6\cdot10^{25}y$ [33] then implies severe limits on any lepton-number violating operator $(\bar{u}d\bar{e}\nu_e^c)$. For scalar couplings one finds [32]

$$G_N[(\bar{u}d\bar{e}\nu_e^c)] \lesssim 10^{-8}\,G_F\,. \tag{4.19}$$

For tensor couplings the constraints are even stronger. The above argument only applies to the operator $(L_1\overline{D}_1)(L_1Q_1)$ that appears in Tab. 2, since the remaining operators also contain leptons of the second or third generation.

Note that one cannot combine two identical NP operators that contain one neutrino to derive a constraint on its coupling, since the resulting $0\nu\beta\beta$ amplitude would be proportional to the neutrino mass, which has no lower bound.

3. Neutrino masses

Lepton number violating operators that include two neutrinos (or two antineutrinos) give rise to neutrino Majorana masses when closing the external charged fermion lines by one or

two loops. If the fermions in the loop have identical flavor a contribution to the neutrino mass is generated at one loop.

Assume that one neutrino ν_i couples to a charged fermion f_k with a mass m_k^f via a scalar singlet (s) or triplet (t) with coupling $\lambda_{ik}^{s,t}$, while the second neutrino ν_j couples to another charged fermion f_l via a scalar doublet (d) with coupling λ_{jl}^d. The mixing of the equal charge components of the two scalar fields gives rise to a L-violating operator as shown in Section II. Let us consider the case where the two charged fermions are identical, i.e. $l = k$. Then the lowest order contribution to the neutrino mass arises at one loop (see Fig. 2). Since the charged fermion propagating in the loop has to flip chirality a mass insertion is needed and the neutrino mass is proportional to m_k^f. The (momentum dependent) propagator for the scalar fields in the loop follows from (2.10) by replacing $M_{1,2}^2 \rightarrow M_{1,2}^2 - p^2$, where p is the loop momentum. Then, the neutrino mass matrix is given by [16]

$$m_{ij}^\nu = iN_c \sum_k (\lambda_{ik}^{s,t}\lambda_{jk}^d + \lambda_{jk}^{s,t}\lambda_{ik}^d) \int \frac{d^4p}{(2\pi)^2} \frac{m_k^f}{(m_k^f)^2 - p^2} \frac{\sin 2\phi}{2} \left(\frac{1}{M_1^2 - p^2} - \frac{1}{M_2^2 - p^2} \right) \quad (4.20)$$

$$\simeq N_c \sum_k \frac{\lambda_{ik}^{s,t}\lambda_{jk}^d + \lambda_{jk}^{s,t}\lambda_{ik}^d}{32\pi^2} m_k^f \sin 2\phi \ln \left(\frac{M_2^2}{M_1^2} \right) , \quad (4.21)$$

where $N_c = 3$ (1) for intermediate quarks (leptons) and the approximation in (4.21) is valid for $m_k^f \ll M_{1,2}$.

How this result can be used to derive constraints on the effective coupling of lepton number violating operators? Connecting two SM beta decays with an intermediate neutrino, one induces $0\nu\beta\beta$ with an amplitude proportional to $G_F^2 \cdot m_{11}^\nu$ and the lower bound on $T_{1/2}^{0\nu\beta\beta}$ translates into a constraint on the m_{11}^ν entry of Majorana mass-matrix. Moreover, assuming that two of the three mentioned neutrino problems are explained by neutrino oscillations (implying $\Delta_{ij} \lesssim 1 \text{ eV}^2$), it follows from unitarity that *all* of the entries of the Majorana mass-matrix can be at most of the order of the upper bound from the Troitsk tritium beta-decay experiment [34] on the lightest neutrino mass eigenstate, $m_1 < 2.5$ eV. So we have

$$m_{ij}^\nu < \begin{cases} 0.36 \text{ eV} & i = j = 1 \\ \sim 3 \text{ eV} & \text{else} \end{cases} . \quad (4.22)$$

Note that if only one of the three neutrino anomalies is explained by neutrino oscillations or if one introduces additional light (sterile) neutrinos [15] the above argument for $i, j \neq 1$ does not hold. However, since our main phenomenological motivation for introducing L-violating interactions is to see whether in such a framework all the three neutrino anomalies could be explained simultaneously with three light neutrinos, we shall use the bounds in (4.22) in the following.

Assuming that there are no significant cancellations between the various terms that contribute to the neutrino mass in (4.21) it follows from (4.22) that

$$\lambda_{ik}^{s,t}\lambda_{jk}^d \sin 2\phi \ln \left(\frac{M_2^2}{M_1^2} \right) \lesssim \left(\frac{\text{MeV}}{N_c m_k^f} \right) \cdot \begin{cases} 6.3 \times 10^{-5} & i = j = 1 \\ 9.5 \times 10^{-4} & \text{else} \end{cases} . \quad (4.23)$$

This implies that the effective coupling of the L-violating operator satisfies:

$$G_N[(\overline{f_{kL}}\,\nu_i)\,(f_{kR}\,\nu_j^c)] = \frac{\lambda_{ik}^{s,t}\lambda_{jk}^d}{8\sqrt{2}}\,\sin 2\phi\left(\frac{1}{M_1^2} - \frac{1}{M_2^2}\right) \tag{4.24}$$

$$\lesssim f(M_2/M_1)\left(\frac{\mathrm{MeV}}{N_c m_k^f}\right)\cdot\begin{cases} 1.9\times 10^{-4}\ G_F & i = j = 1 \\ 2.9\times 10^{-3}\ G_F & \text{else} \end{cases}, \tag{4.25}$$

where

$$f(x) \equiv \frac{1 - x^{-2}}{\ln x^2} < 1 \quad \text{for } x > 1\,, \tag{4.26}$$

and we have set the lower mass $M_1 = 50$ GeV to its minimal value. We learn that the above constraints on G_N in many cases are stronger than those listed in the third and fourth section of Tab. 2. In particular, all the effective couplings for lepton number violating scattering off electrons and quarks are at most of the order $6\times 10^{-3}\ G_F$ and a few $\times 10^{-4}\ G_F$, respectively.

V. LEPTON NUMBER VIOLATING INTERACTIONS AND NEUTRINO OSCILLATION EXPERIMENTS

Having introduced and motivated the L-violating interactions induced by scalar mixing, we turn now to a systematic survey of those interactions that are relevant to the terrestrial, solar and atmospheric neutrino experiments.

A. Terrestrial neutrino experiments

The LSND collaboration has reported a positive signal in two different appearance channels. The first analysis [4] uses $\bar{\nu}_\mu$'s from muon decay at rest (DAR) and searches for $\bar{\nu}_e$'s via inverse beta decay. The observed excess of $\bar{\nu}_e$ events corresponds to an average transition probability of [4]

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = (3.1^{+1.1}_{-1.0} \pm 0.5)\times 10^{-3}. \tag{5.1}$$

Explaining this result in terms neutrino oscillations, requires Δm^2 and $\sin^2 2\theta$ in the range indicated in Fig. 3 of Ref. [4]. Taking into account the restrictions from the null results of other experiments, the preferred values of the neutrino parameters are $\Delta m^2 \approx 2$ eV^2 and $\sin^2 2\theta \approx 2\times 10^{-3}$ and the *lower* limit on Δm^2 for the neutrino oscillation solution is given by

$$\Delta m^2 > 0.3 \text{ eV}^2\,. \tag{5.2}$$

The second analysis [5] uses ν_μ's from pion decay in flight (DIF) and searches for ν_e's via the $\nu_e\,C \to e^-\,X$ inclusive reaction. Again a positive signal with a transition probability

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$P(\nu_\mu \to \nu_e)$ similar to the one in (5.1), but with less statistical significance, has been reported.

Besides the orthodox neutrino oscillation hypothesis it has been proposed that the LSND signals could be due to non-standard neutrino interactions [19–21]. Assuming that the result in (5.1) is due to New Physics interactions of strength G_N^ν (while there is no significant contribution from neutrino oscillations) the appearance probability is given by [19]

$$P(e^+) = \left| \frac{G_N^\nu}{G_F} \right|^2 . \tag{5.3}$$

From eqs. (5.1) and (5.3) we learn that, in order to explain the LSND result, the effective NP coupling should satisfy

$$G_N^\nu > 4.0 \times 10^{-2} \, G_F \tag{5.4}$$

at the 90% confidence level (CL). In Ref. [25] it has been shown that lepton flavor violating neutrino interactions cannot satisfy the condition (5.4) even if one allows for maximal $SU(2)_L$ breaking effects.

Here we investigate whether lepton number violating interactions could be large enough to be relevant for the LSND results. As we already mentioned in the Introduction the anomalous L-violating decays (1.4) could in principle be a possible source for the $\bar\nu_e$'s in the DAR channel of LSND. From the first section of Tab. 2 it follows that these decays can be mediated by a scalar doublet that couples to $A^\dagger = L_1 \overline{E}_2$ whose charged component mixes with a scalar singlet that couples to $B = L_1 L_\ell$. (Here $\ell = \mu, \tau$, but for a scalar triplet also $\ell = e$ is possible.) Comparing the model-independent bounds for the effective couplings $G_N^{A^\dagger B} \lesssim 0.02 \, G_F$ with the required effective coupling strength in (5.4) we conclude that in the $SU(2)_L$ symmetric case $G_N^{A^\dagger B}$ is too small to explain the LSND DAR result. However, while $SU(2)_L$ breaking effects cannot be large, an enhancement by a factor of two, which is required to satisfy (5.4), is indeed conceivable. Thus we cannot rule out in a model independent way that the lepton number violating decays in (1.4) are the source of the LSND anomaly. However, moving to the explicit framework of SUSY $\rlap{/}R_p$ the constraints on $G_N^{A^\dagger B}$ are stronger by a factor of four implying that even with maximal $SU(2)_L$ breaking one cannot fulfill (5.4), unless one allows for some fine-tuned cancellations.

It is interesting to ask whether the L-violating interactions

$$\nu_\ell \, p \to e^+ \, n \,. \tag{5.5}$$

could provide an alternative explanation for the DAR signal. As one can see from the second section of Tab. 2 the processes in (5.5) can be induced by scalar mixing if either $A^\dagger = L_\ell \overline{D}_1$ to $B = Q_1 L_1$ or $A'^\dagger = Q_1 \overline{D}_1$ to $B' = L_1 L_\ell$. While the scalar fields coupling to A and A' have to be doublets, those coupling to B and B' could be either singlets or triplets of $SU(2)_L$. (Note that if $L_1 L_\ell$ couples to a singlet then this excludes $\ell = e$ due to the antisymmetry of the singlet contraction.) However, as we have noted in Section IV B 1, any operator that induces

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the reaction in (5.5) also necessarily gives rise to lepton number violating pion decays. Thus, the stringent constraints in (4.15) and (4.16) apply, unless one is willing to allow for a fine tuned cancellation in (4.14) by setting $G_N^e/m_e = G_N^\mu/m_\mu$. But even in this case according to the bound in Tab. 2 the effective coupling $G_N^{A^\dagger B}$ of the operator $A^\dagger B$ is much too small to satisfy (5.4). Using only the bounds from the trilinear couplings in Tab. 2 $G_N^{A'^\dagger B'}$ could be consistent with (5.4) provided that there is an enhancement from $SU(2)_L$ breaking effects by a factor of two. The corresponding bound within SUSY $\not{R}_p$ is only stronger by a factor of three. Although it is rather unlikely, we cannot rule out completely that the lepton number violating reactions in (5.5) play a role for the LSND DAR result.

We note that lepton number violating pion decays $\pi^+ \to \ell^+ \bar{\nu}_e$ cannot be responsible for the $\bar{\nu}_e$'s observed by LSND, even though they are not helicity suppressed. First, according to (4.11) the BR for $\ell = e$ is *measured* to be too small. Second, for $\ell = \mu$ the kinetic energy of the final $\bar{\nu}_e$ is at most 34 MeV, which is below the threshold energy of the LSND DAR analysis.

As concerns the LSND DIF channel an interpretation of this anomaly in terms of L-violating interaction is less attractive for the following reason: The presence of additional L-violating pion decays of the form $\pi^+ \to \mu^+ \bar{\nu}_\ell$ cannot produce the observed ν_e's. Likewise L-violating interactions in the detection process could only imply that neutrons capture antineutrinos which are absent in the SM pion DIF. Hence the (generically suppressed) L-violation processes would be required for both the neutrino production and detection, ruling out this scenario as an explanation for the LSND DIF signal.

We note that the KARMEN experiment [6], which uses the same detection processes as LSND has found no evidence for neutrino flavor transitions ruling out a transition probability as in (5.1) at 90% CL. In general this situation somewhat favors an explanation of the LSND anomaly in terms of "standard" neutrino oscillations, since due to the different baselines there is still as small region in the $\Delta m^2 - \sin^2 2\theta$ plane consistent with both experiments, while for new physics reactions as the source of the LSND anomalies KARMEN should observe the same transition probabilities. Still, the bound from KARMEN gains from the fact that less events were observed than expected from the background, so for conclusive evidence we will have to wait for the upcoming MiniBooNE experiment [9] (see also Section V D).

A different search for neutrino oscillations has been performed by the CHORUS [35] and NOMAD [36] experiments at CERN looking for $\nu_\mu \to \nu_\tau$ oscillations transitions. In the absence of neutrino flavor transitions the relative flavor composition of the neutrino beam is predicted to be $\nu_\mu : \bar{\nu}_\mu : \nu_e : \bar{\nu}_e = 1.00 : 0.061 : 0.0094 : 0.0024$ with a negligible ($\simeq 10^{-7}$) contamination of tau neutrinos. The search for ν_τ is based on charged current tau production with subsequent detection of the various tau decay modes. Both experiments have found no indication for $\nu_\mu \to \nu_\tau$ oscillation. The upper bound from NOMAD [36] on the transition probability is

$$P(\nu_\mu \to \nu_\tau) < 0.6 \times 10^{-3} \quad (90\% \; CL) \,. \tag{5.6}$$

Since also the observed τ^+ events are in agreement with the estimated background a similar bound as in (5.6) applies to $P(\nu_\mu \to \bar{\nu}_\tau)$. The production of τ^+'s could also be induced by the lepton number violating reaction

$$\nu_\mu \, p \to \tau^+ \, n \,, \tag{5.7}$$

which would result from the operators $(L_2 \overline{D}_1)(L_3 Q_1)$ or $(Q_1 \overline{D}_1)(L_2 L_3)$ that appear in Tab. 2. It is interesting to note that the upper bounds we obtained for the effective coupling of these operators in Section IV (see Tab. 2) are of the same order $[(\hat{G}_N / G_F)^2 \sim 10^{-3}]$ as the experimental constraint from NOMAD (and CHORUS). Unfortunately the proposed TOSCA experiment [37] that would have been sensitive to a transition probability as small as $\sim 10^{-5}$ has been rejected.

B. Solar neutrino experiments

The long standing solar neutrino puzzle [3] is now confirmed by five experiments using three different experimental techniques and thus probing different neutrino energy ranges. All these experiments observe a solar neutrino flux that is smaller than expected. The most plausible solution is that the neutrinos are massive and there is mixing in the lepton sector. Then neutrino oscillations can explain the deficit of observed neutrinos with respect to the Standard Solar Model. In the case of matter-enhanced neutrino oscillations, the famous MSW effect provides an elegant solution [3] to the solar neutrino problem with Δ_{SN} as given in (1.1).

Several authors have studied alternative solutions to the solar neutrino problem with and without neutrino masses [17,22]. In the scenario with massive neutrinos Δ_{SN} is still required to be of the same order as in (1.1). However, the vacuum mixing can be vanishingly small [22], when the effective mixing is dominantly induced by the flavor changing neutrino scattering

$$\nu_e \, f \to \nu_\ell \, f, \tag{5.8}$$

where $\ell = \mu, \tau$ and $f = e, u, d$.

For the scenario without neutrino masses additional non-universal flavor diagonal interactions

$$\nu_\ell \, f \to \nu_\ell \, f, \tag{5.9}$$

are required. For both scenarios the effective couplings in (5.8) $G^f_{e\ell}$ have to be of the order of a few percent, while for the second scenario the difference between the effective couplings in (5.9) $G^f_{\ell\ell} - G^f_{ee}$ has to be in the narrow interval $[0.50\,G_F, 0.77\,G_F]$ ($[0.40\,G_F, 0.46\,G_F]$) for $f = d$ (u) to allow for a resonant neutrino conversion. This requires rather large non-universal flavor diagonal couplings, which is ruled out for $\ell = \mu$ [23].

It is interesting to ask whether also the L-violating neutrino scattering

$$\nu_e f \to \bar{\nu}_\ell f, \tag{5.10}$$

where $\ell = \mu, \tau$ and $f = e, u, d$ in combination with either massive neutrinos (but negligible mixing) or additional non-universal flavor diagonal interactions of the type

$$\bar{\nu}_\ell f \to \bar{\nu}_\ell f, \tag{5.11}$$

could give rise to matter-induced $\nu_e - \bar{\nu}_\ell$ neutrino oscillation that provide an alternative solution to the solar neutrino problem. Note that the L_ℓ and L conserving interactions in (5.9) and (5.11) are related by crossing symmetry. So their effective couplings are subject to the same bound.

As we have seen in Section II lepton number violating reactions as in (5.10) require mixing between intermediate bosons with different $SU(2)_L$ transformations. The third section of Tab. 2 contains various combinations of bilinears that when coupled to each other by scalars that mix can induce the L-violating neutrino scattering off a fermion as in (5.10). The total lepton number will only be violated if the two bilinears $A_{q_f} = \bar{\nu} f_A$ and $B_{q_f} = \nu f_B$ contain charged fermions f_A and f_B that belong to different presentations of $SU(2)_L$. Consequently, independent of the details of the model, the reordered four-fermion operator that induces the effective neutrino potential

$$\mathcal{H}_{\text{int}} = \frac{G_F}{\sqrt{2}} \sum_{a=S,P,T} (\overline{\nu^c}\, \Gamma^a\, \nu) \left[\overline{\psi}_f\, \Gamma_a \left(g_a + g'_a \gamma^5 \right) \psi_f \right] + \text{h.c.} \,, \tag{5.12}$$

can only contain scalar ($\Gamma^S = I$), pseudo-scalar ($\Gamma^S = \gamma_5$) or tensor ($\Gamma^T = \sigma^{\mu\nu}$) couplings. (Axial)vector couplings are not possible, since they couple between fermions of the opposite chirality.

To be explicit consider the example within SUSY $\not{R}_p$ where the $q = 1/3$ component of $A^\dagger = L_3 \overline{D}_1$ may couple to the weak singlet $B = L_1 Q_1$ if there is $\tilde{b}_L - \tilde{b}_R$ mixing. The resulting four-fermion operator is:

$$\mathcal{H}_{\text{int}} = \frac{\lambda'_{331} \lambda'_{313}}{M^2_{AB}} (\overline{d_R}\, \nu_\tau)(\overline{\nu_e^c}\, d_L) = -\frac{\lambda'_{331} \lambda'_{313}}{M^2_{AB}} \left[\tfrac{1}{2}(\overline{\nu_e^c}\, \nu_\tau)(\overline{d_R}\, d_L) + \tfrac{1}{8}(\overline{\nu_e^c}\, \sigma_{\mu\nu}\, \nu_\tau)(\overline{d_R}\, \sigma^{\mu\nu}\, d_L) \right] \,. \tag{5.13}$$

The question is then whether scalar and tensor couplings can affect the neutrino propagation in dense matter significantly. The bounds from the neutrino masses in (4.25) indicate that the relevant effective couplings G_N are less than $10^{-2(-3)}\, G_F$ for neutrino scattering of electrons (quarks). This constraint could be evaded if we only accept one measurement for mass squared difference Δm^2 and allow for one mass eigenstate much heavier than 2.5 eV. However, we still have the bounds from the trilinear couplings (see Tab. 2), which imply that G_N is at most at the few percent level. Then the lepton number violating interactions could

only affect the standard MSW oscillations if the averaged matrix element of the background fermion current in (5.12) is of similar order as the one from the SM weak current [22]. In Ref. [38] it has been shown that for (pseudo)scalar interactions the effective neutrino potential that is induced by (5.12) is proportional to the ratio of the neutrino mass and the characteristic fermion energy. Thus (pseudo)scalar couplings in (5.12) are not relevant for matter induced neutrino oscillations in the Sun. Moreover, it has been pointed out that transverse tensor couplings are not suppressed by the neutrino mass [38]. However, in this case the effective neutrino potential is proportional to the average (transverse) polarization of the background matter. Since the polarization in the solar interior due to the magnetic field is expected to be tiny [38], we conclude that the L violating neutrino scattering in (5.10) is not relevant for the solar neutrino problem.

C. Atmospheric neutrino experiments

Several experiments have observed an anomalous ratio between the atmospheric muon neutrino and electron neutrino fluxes [2]. This atmospheric neutrino problem has recently been confirmed by the Super-Kamiokande high statistics data [1]. Explaining this result in terms of "standard" neutrino oscillations [2] requires a mass squared difference Δ_{AN} as shown in (1.2).

Recently an alternative solution to the atmospheric neutrino anomaly based on new neutrino interactions was proposed [18]. The suggested scenario is similar to the one we discussed previously for the solar neutrino, but for $\nu_\mu - \nu_\tau$ oscillations. Even if neutrino masses are negligible the effective mixing between the flavor eigenstates could in principle be induced by the flavor changing neutrino scattering

$$n_\mu\, f \to n_\tau\, f, \tag{5.14}$$

where $n = \nu, \bar{\nu}$ and $f = e, u, d$, in combination with non-universal flavor diagonal interactions

$$n_\ell\, f \to n_\ell\, f, \tag{5.15}$$

with $\ell = \mu, \tau$. According to [18] the effective couplings $G_{\mu\tau}^f$ for (5.14) and the difference between the effective couplings for (5.15), $G_{\tau\tau}^f - G_{\mu\mu}^f$, have to be both of order $0.1\, G_F$. As has been shown in Ref. [24] $G_{\mu\tau}^f$ is constrained by electroweak precision data to be at most at the few percent level ruling out such an explanation, unless one allows for some fine-tuned cancellations.

However, in view of the bounds in the fourth section of Tab. 2 on the effective couplings for the lepton number violating neutrino scattering

$$\nu_\mu\, f \to \bar{\nu}_\tau\, f \quad \text{and} \quad \bar{\nu}_\mu\, f \to \nu_\tau\, f, \tag{5.16}$$

one might wonder whether such interactions could offer an alternative mechanism to solve the atmospheric neutrino anomaly. Although the effective coupling for $(L_3 \overline{D}_1)(L_2 Q_1)$ could

be of order $0.1\,G_F$, such an explanation faces the same problem that we encountered in the discussion of solar neutrinos. Namely, the inherent change of the chirality of the background fermions restricts the couplings to be of scalar or tensor type. Consequently the effective neutrino potential is suppressed by either the neutrino mass or the average polarization. Thus we conclude that lepton number violating interactions do not effect atmospheric neutrinos that propagate through earth matter.

D. Future terrestrial neutrino oscillation experiments

The fundamental difference between neutrino transitions induced by new interactions and "standard" vacuum neutrino oscillations due to non-vanishing neutrino masses and mixing is that only the latter have a non-trivial L/E (distance over energy) dependence if the neutrinos propagate in vacuum. Flavor changing neutrino interactions that conserve total lepton number in principle can induce matter-induced neutrino oscillations that are distance dependent. Among laboratory neutrinos matter effects are only relevant for long baseline experiments, where the neutrinos propagate through the earth mantle [39]. However, for $\nu_\mu \to \nu_\tau$ transitions the flavor changing parameter ϵ has be of order unity [40] to be relevant for the K2K [11] and MINOS [12] long baseline neutrino experiments, which is inconsistent with the model-independent bounds presented in Refs. [24]. Also for $\nu_e \to \nu_\mu(\nu_\tau)$ transitions ϵ is at most of order $10^{-5}(10^{-2})$ [23] implying that earth effects will not probe the flavor changing interactions in the upcoming long baseline detectors.

As we pointed out in our discussion of solar neutrinos in Section V B matter effects on the neutrino propagation due to lepton number violating interactions are even less significant due to the suppression from the neutrino mass or the background polarization. Therefore the dominant impact on terrestrial neutrino experiments from new interactions comes from the modification of the relevant detection and/or production processes, like the reaction in (1.4) that we discussed for LSND. This fact allows us to distinguish the proposed solution of LSND in terms of lepton number violating interactions without any theoretical assumptions, just on the basis of the experimental observations. In the future a number of terrestrial neutrino experiments with different baselines will try to clarify the nature of neutrino oscillations. Should a certain neutrino transition channel maintain a distance independent contribution (beyond the trivial decrease of the flux inverse to the distance squared) this would signal non-standard neutrino interaction. In the following we discuss briefly some of the upcoming experiments and their potential to observe lepton number violating interactions.

The MiniBooNE experiment [9] at Fermilab is designed to confirm (or refute) the $\nu_e - \nu_\mu$ oscillations signals observed at LSND by searching for $\nu_\mu \to \nu_e$ transitions with an expected sensitivity to $P(\nu_\mu \to \nu_e) > 2 \cdot 10^{-4}$ at 90% CL. Also a search for $\bar\nu_\mu \to \bar\nu_e$ seems feasible even though $\bar\nu_\mu$'s are produced less copiously (by a factor ~ 0.2). The important feature of MiniBooNE is its small background of ν_e and $\bar\nu_e$, which allows to search also for the lepton number violating transitions $\nu_\mu \to \bar\nu_e$ and $\bar\nu_\mu \to \nu_e$ with a sensitivity of a few times

10^{-4}. Therefore it might be possible to probe the operator $(Q_1\overline{D}_1)(L_1L_2)$ at the level of its model-independent bound obtained in Section IV (c.f. Tab. 2).

Several long baseline experiments [10] will search for neutrino transitions in particular aiming at the atmospheric region of evidence for $\nu_\mu \to \mu_\tau$ oscillations. While the long baseline allows these experiments to explore small mass-squared differences $\Delta m^2 \gtrsim 10^{-3}\,\mathrm{eV}^2$, the neutrino flux at the detector of the upcoming experiments is rather small yielding at most a few hundred events per year. Therefore we do not expect that these experiments could probe lepton number violating interactions anywhere close to the bounds obtained in Section IV (c.f. Tab. 2). However it might be possible to obtain interesting information from a second detector very close to the neutrino source, which is supposed to study the initial neutrino beam.

E. Supernova neutrinos

While the effects from lepton number violating neutrino scattering as in (5.10) and (5.16) are negligible for solar and atmospheric neutrinos, these reactions could be relevant for the neutrinos emerging from a supernova explosion. Here tensor interactions could affect the neutrino propagation provided that there is a very large magnetic field $B \sim 10^{16}\,\mathrm{G}$ which induces a large polarization $\langle \lambda_f \rangle \simeq 10^{-2} - 10^{-1}$ [38].

VI. CONCLUSIONS

We have presented a comprehensive analysis of lepton number violating neutrino interactions. L-violating four-fermion operators involving one or two neutrinos are induced by heavy boson exchange, if there is mixing between the equal charge components of a doublet and a singlet (or a triplet) of $SU(2)_L$.

As an example we have discussed SUSY $\not{R}_p$, where such operators are induced by the mixing of "right-handed" sfermions that are $SU(2)_L$ singlets with the "left-handed" sfermions that are $SU(2)_L$ doublets.

We have studied four approaches to constrain the L-violating operators in a model-independent framework:

1. Constraints from the trilinear couplings,

2. Universality in pion decays,

3. Neutrinoless double beta decay,

4. Loop-induced neutrino masses.

Any non-vanishing coupling between any fermionic bilinear and a scalar field induces an effective four-fermion operator containing the bilinear and its hermitian conjugate with an

effective coupling proportional to the square of the trilinear coupling over the scalar mass. Combining the upper bounds on such effective operators one can derive constraints on the lepton number violating operators that consist of two different bilinears.

The measured ratio between the BR for pion decays with final electrons and those with final muons is in good agreement with the value predicted by the SM. This implies severe constraints on NP contribution to these pion decays. Since the final neutrinos are not observed this includes lepton number violating pion decays. Moreover, since these decays are not helicity suppressed, there is a significant enhancement from the hadronic matrix element, which gives rise to stringent limits on the relevant effective couplings.

Any L-violating four-fermion operator that can be combined with the SM operator responsible for beta decay in order to induce neutrinoless double beta decay is severely constrained by the experimental limit on this process. Unlike for the "standard" neutrinoless double beta decay, where the lepton number violation is induced by the neutrino Majorana mass, in the New Physics case, L is broken by the operator and the intermediate neutrino can contribute by its momentum rather than its mass.

Lepton number violating operators containing two neutrinos and two charged fermions of identical flavor (but opposite chirality) induce neutrino Majorana masses at one loop when the external fermions lines are connected by a fermion propagator (see Fig. 2). Using the upper bound on the lightest neutrino mass-eigenstate from the Troitsk tritium experiment and assuming a three neutrino framework with mass-splittings not larger than ~ 1 eV, we derive stringent constraints on the relevant lepton number violating operators.

Our constraints lead to the following conclusions:

- Lepton number violating neutrino scattering $\nu_\alpha f \to \bar{\nu}_\beta f$ off matter fermions $f = e, u, d$ are severely suppressed by the bounds from neutrino masses and do not play a role for the present solar and atmospheric neutrino anomalies. Since any effect due to such interactions would require also a polarized background, it could – at best – be relevant for supernova neutrino oscillations.

- Model-independent considerations show that only the operators $(L_1 \overline{E}_2)(L_1 L_{2,3})$ (inducing the anomalous muon decay $\mu_L^+ \to e_R^+ \bar{\nu}_e \bar{\nu}_{\mu,\tau}$) could have an effective coupling at the few percent level (of G_F) and thus might be significant for the LSND anomaly. Lepton number violating neutrino capture by protons is severely constrained by the data on pion decays, and not relevant for LSND, unless one is willing to accept some fine-tuned cancellations. Within SUSY $\cancel{R}_p$ the relevant constraints are stronger by a factor of four, and an explanation of the LSND DAR data via lepton number violating interactions is inconsistent with the upper bound on the maximal $SU(2)_L$ breaking.

A solution of LSND in terms of New Physics is attractive since this way the solar and the atmospheric neutrino anomalies could be explained via standard neutrino oscillations avoiding the introduction of a sterile neutrino. However, this interpretation seems to be

somewhat disfavored by the confirmation of the LSND anomaly in the DIF data and by the null signal of KARMEN.

Future terrestrial neutrino oscillation experiments that are sensitive to the "neutrino transition" probability at the level of 10^{-4} could observe lepton number violating neutrino interactions. Any signal that does not depend on the baseline is a potential candidate for new neutrino interactions. However, the present solar and atmospheric neutrino anomalies are not (significantly) "contaminated" by such interactions.

ACKNOWLEDGMENTS

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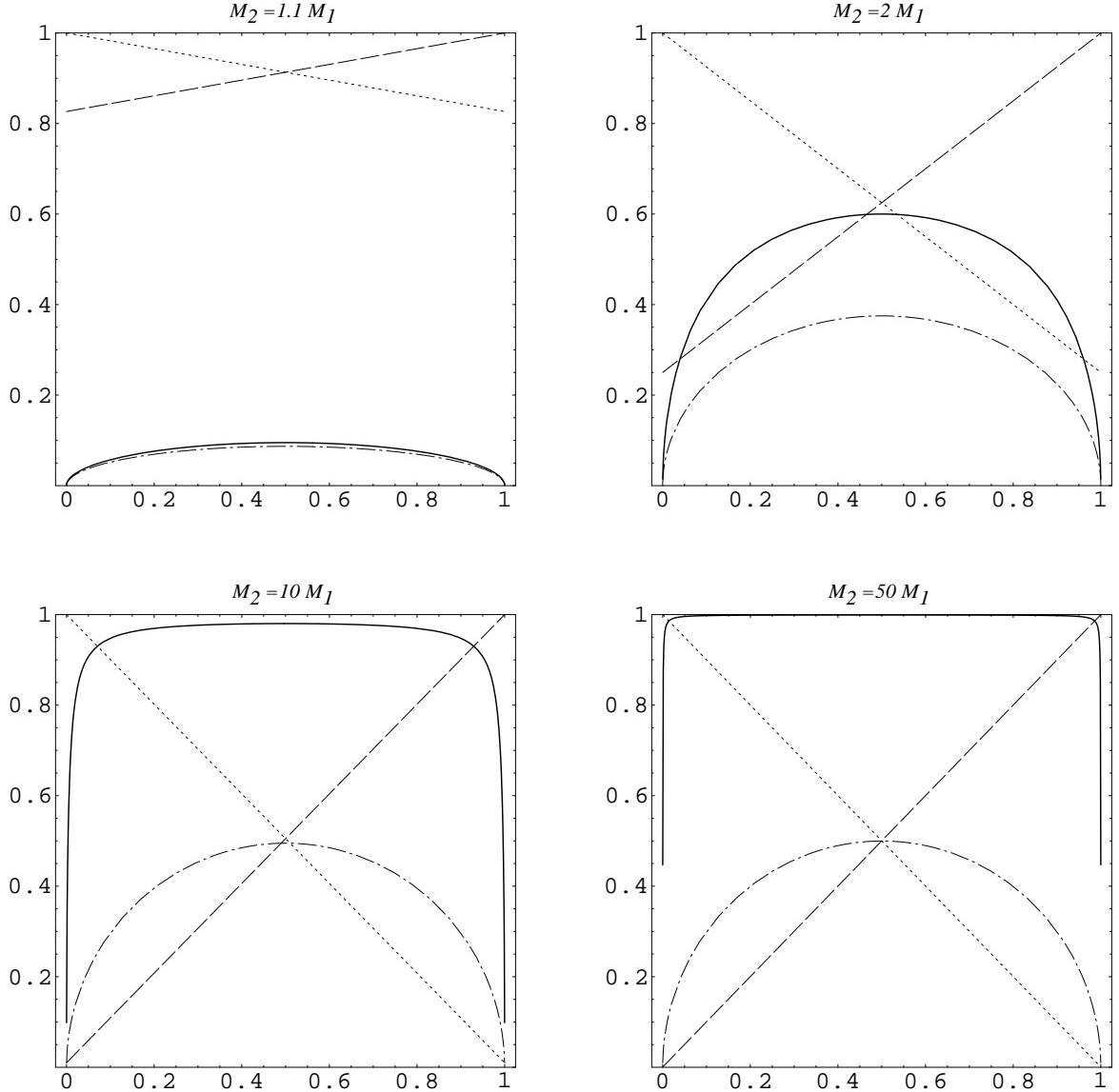
$\sin^2\theta$

FIG. 1. The solid curve shows the suppression factor $\varrho = \frac{M_A M_B}{M_{AB}^2}$ (horizontal axes) as a function of the $\phi - \chi$ mixing $\sin^2\theta$ (vertical axes) for $M_2/M_1 = 1.1, 2, 10, 50$. Also the dependence on $\sin^2\theta$ for the propagators M_A^{-2}, M_B^{-2} and M_{AB}^{-2} (in units of M_1^{-2}) is indicated by the dotted, dashed and dashed-dotted curves, respectively.
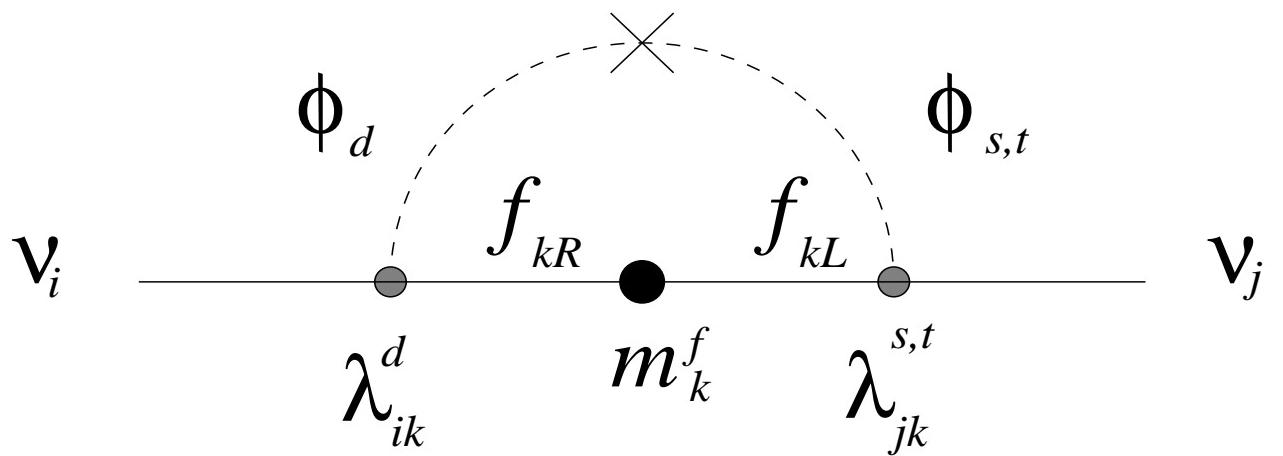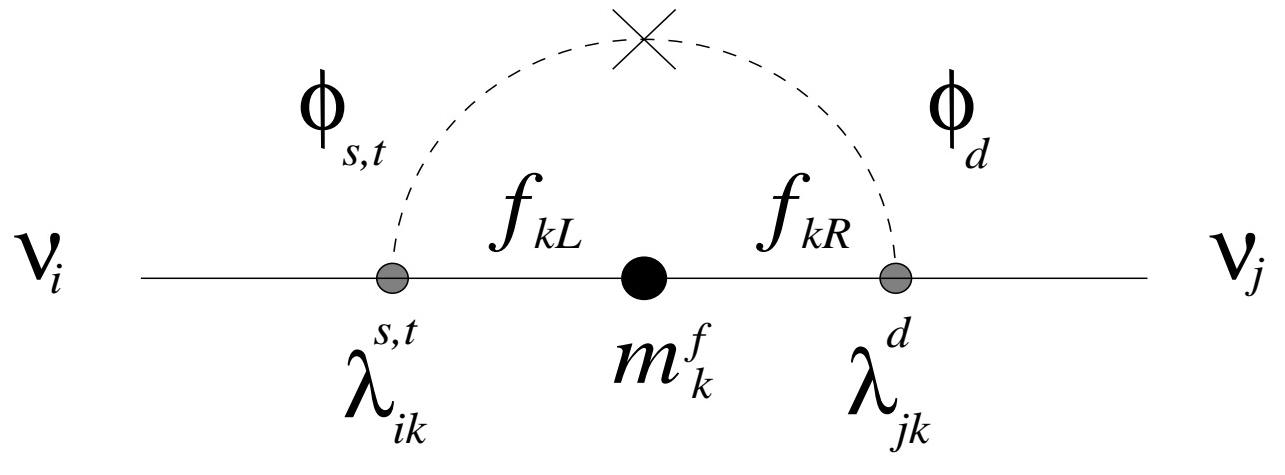
FIG. 2. Feynman diagram for neutrino Majorana mass term m_{ij}^{ν} induced at one loop.